Reply to ''Comment on 'Corresponding states of periodic structures in nematic liquid crystals' ''

A. J. Palangana, M. Simões,* L. R. Evangelista, and A. A. Arrotéia
Departamento de Física, Universidade Estadual de Maringà, Avenida Colombo, 5790, 87020-900 Maringá, Paraná, Brazil
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We reply to the Comment of Grigutsch and Stannarius [Phys. Rev. E (to be published)] on our paper, Palangana *et al* [Phys. Rev. E 56, 4282 (1997)]. The fact that a viscous process determines the geometry of the magnetic walls of a nematic liquid crystal sample is used to show that the viscosity coefficients appear through its relative values. These values are very similar in several different nematic samples. This similarity is proposed as being responsible for the existence of the corresponding states. [S1063-651X(99)08207-0]

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We thank the authors of the Comment [1], Grigutsch and Stannarius (GS) for the attention devoted to our paper [2]. Let us present a summary of their criticism. In our paper, some macroscopic parameters of the walls appearing above the Fréedericksz threshold have been scaled, and we have found that the resulting distribution of points suggests the existence of a universal line of corresponding states. GS affirm that the use of a wall to exemplify our corresponding states law is incorrect. We have used a static model to deduce the scaling rules, and the walls result from a dynamic selection mechanism, involving the viscosity and the elasticity of the system. Furthermore, according to GS, two samples differing only by the viscosity coefficients, but having identical elastic constants, will yield curves with different wavelengths.

We agree with GS when they affirm that the full explanation of the law of the corresponding states cannot be static but requires a dynamic approach. But our paper does not contain the affirmation that the arising of the magnetic walls can be fully understood in the framework of a static approach. This fact is explicitly recognized, for instance, on p. 4284 of the commented paper, where we have stressed that ''... in order to find this dependence it would be necessary to consider the physics of the system in the instant in which the walls were created, which would involve the study of a nonlinear process, and that is not our aim here. We want just to *describe* the existence of the corresponding states...'' Furthermore, a few lines ahead, we have recognized that the static considerations cannot give an explanation for the law of the corresponding states and affirm: ''... Certainly, in order to achieve such a requirement, each particular system could give a particular value to these parameters. We have found not only that all the experimental points can indeed be put along the same universal line...'' Finally, at the Conclusion we repeat ''... This constant must be determined by the physics of the system in the moment in which the walls are produced...'' Therefore, the commented paper does not intend to give a theoretical explanation of the law of the corresponding states itself, but only to expose it as an experimental achievement. It must be emphasized that the static theory was used for two purposes: (i) to give a universal equation,

$$\partial_x^2\eta-\eta+2h^2u'(\eta)=0, \tag{1}$$

describing the walls' shape, and (ii) to give the scaling laws

$$\chi_a H_c^2=K_{33}\left(\frac{\pi}{b}\right)^2+K_{22}\left(\frac{\pi}{d}\right)^2,\quad x^2=\frac{K_{33}}{\chi_a H_c^2}\overline{x^2},\quad h=\frac{H}{H_c}, \tag{2}$$

leading to the above equation and to the corresponding states (the definition of the parameters of the above equations can be found in Ref. [2]).

The scaling rules given by Eq. (2) lead to Eq. (1) and would be applicable for static configurations (see Refs. [2,3]). Notwithstanding, we used it in the walls that have been produced by a dynamical mechanism. Before the application of above scaling rules the measured points are spread, and there is not any evident correlation among them. But, as can be observed in Ref. [2], after the application of scaling rules they look as if there was a sole line along which they coalesce. As observed by GS, this result is unexpected. The elastic properties of the nematic liquid crystals (NLC) material are not the unique factors responsible for the geometry of the walls. Lonberg *et al*. [4] have shown how the combined action of the external field on the director and the fluid motion produce the walls. Therefore, it may be expected that some dynamic parameters, such as the viscosity coefficients, will contribute to the determination of their geometrical properties. This contribution should emerge through the constants of integration of Eq. (1). More precisely, even being the measurements of the walls' parameters made in a static condition, the constants of integration in Eq. (1) are fixed

*Permanent address: Departamento de Física, Universidade Estadual de Londrina, Campus Universitário, Londrina, Paraná, Brazil.

during the fluid flow that gives rise to the walls. Therefore, some information about the physcial conditions prevailing in its origin is expected to be found in the parameters of the walls. It is with some surprise that one discovers that only the static parameters, through the scaling presented above, are enough to put the experimental data along a single curve. Consequently, the success of the scaling laws, in reducing these experimental data to a possible universal line, seems to indicate that there is also some unknown uniformity in the dynamic process that has built the walls.

Since the publication of our paper, we have worked on a dynamical justification for the law of corresponding states. A summary of the conclusions of this paper will be given below, the full explanation has appeared in Ref. [3]. By considering the dynamic process of creation of the walls we were able to show that the relative values of the viscosity coefficients are the dynamical parameters determining the wavelength of the periodic structure. Due to that, samples differing only on the viscosity coefficients—but having similar reduced values—may produce curves very similar. To our surprise the data collected in NLC literature concerning different compounds presented an approximated similarity among the relative values of the viscosity coefficients. The calculations done in Ref. [3] indicate that the key to understanding this unexpected behavior of the walls' corresponding states is this coincidence in the reduced viscosity coefficients.

One interesting point emphasized by GS concerns our deduced form for the walls. In a previous work [2] we have shown that, in a fourth-order polynomial approximation for the free energy, the shape of the walls would be given by an elliptic function, whose form factor k fixes the shape of the walls that would be determined by the external magnetic field, $k=k(h)$. In our work an *ad hoc* form was assumed for this equation. Two interesting observations made by GS about this assumed form are that our relation is not in accord with the experimental data, and that the proper relation can be numerically deduced from the Lonberg theory. Moreover, they claim that the results exhibited in Fig. 3 of their Comment are in disagreement with our *ad hoc* relation.

First of all, it is important to stress that the core of their critique resides in Eq. (3) of their paper. That relation follows from the work of Lonberg *et al*. [4] and shows how the walls periodicity is selected as a function of the external magnetic field. Even being an important relation in the history of the phenomenology of these walls, it is not in accord with the experimental observations. We are not the only ones to affirm it. This fact is well known in the NLC literature. To quote a few works, we point out that Srajer, Fraden, and Meyer [5] showed that, in order to be in accord with the experimental data, this relation needs some improvements (notice that Fraden and Meyer are also co-authors of the original work of Lonberg). One of the changes made by them was the introduction of nonlinear effects in the selection mechanism of the walls' periodicity. This is exactly what we have done in our justification for the corresponding states [3]. Amengual *et al*. [6], in one of the papers quoted by GS, also have made a detailed study of the nonlinearity of this selection mechanism, and from their results also follows that Eq. (3) of [1] needs improvements. Even GS, in a previous work [7], recognize the existence of this nonlinear effect.

Nevertheless, if we accept the result of their Fig. 3 as true, for all values of the magnetic field one will find $k(h)>0.92$. Furthermore, for both limits, $h^2\rightarrow 1^+$ and $h^2\rightarrow\infty$, one would obtain $k(h)\rightarrow 1$. As we have demonstrated [2], the parameter k governs the shape of the walls, and when $k\rightarrow 1$ the wall would be given by an abrupt, infinitely thin, transition between the two stable configurations. This wall would be described by the hyperbolic tangent where the saturated portion of the wall is its dominant portion [8]. Therefore, if we accept the result presented by GS, as the Frèedericksz threshold is approached the wall will become more abrupt, and its saturated portion will become much larger. This has not any experimental evidence. Léger [9] has measured the length of this transition and has shown that it goes exactly the opposite way. As the Fréedericksz transition is approached, it becomes larger and larger. So, is it not a absurd by itself to believe that in both limits, $h^2\rightarrow 1^+$ and $h^2\rightarrow\infty$, the walls shape would be the same? In the limit of $k\rightarrow 0$, the wall would be given by an infinitely large sinusoidal profile [10] (it is enough to observe that when $h^2\rightarrow 1^+$ the dominant term in the polynomial expansion of the free-energy density would be the quadratic one [8]). Therefore, as $h^2\rightarrow 1^+$, we find $k\rightarrow 0$.

In the GS argument a relation is used that follows from a linear analysis. As it has been demonstrated in the paper of Srajer *et al*. [5], Amengual *et al*. [6], and quoted in a paper of GS [7], when the nonlinear analysis is used, a shift of the walls' periodicity is observed. This means that the function $k(h)$ would be shifted to small values. Therefore, while the relation used by GS is important to understand the origin of the selection mechanism for the periodicity of the walls, it cannot be considered as the final result when experimental values are considered, because nonlinear effects have to be taken into account. For example, as we have shown above, one would expect that when $h^2\rightarrow 1$ one would have $k\rightarrow 0$, and not $k\rightarrow 1$. It was this reasoning that led us to Eq. (4) of [1]. The simplest relation making the transition between $k=0$ and an asymptotic value of $k=1$ is an exponential. Surely, the correctness of this relation must be proved. But, in any way, it cannot result from the linear analysis proposed by the authors of the Comment.

To conclude, we have applied the same scaling that leads the static equations to a universal equation, to the dynamic equations describing the arising of the walls. It was discovered that the resulting equations have no more elastic parameters. The viscosity coefficients still remain in these equations as a ratio. We have found from the literature data that this ratio is much more coincident than the absolute values of the coefficients and this coincidence is the reason why our points seem to be distributed along a single line. Therefore, as this reduced viscosity are neither scaled out of the dynamic equations or absolutely equal, we must be rigorous and affirm that this line of corresponding states is, by now, a coincidence. But, we believe that once this coincidence is justified on theoretical grounds, this fact can justify the existence of the corresponding states.

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